

## Sequences

## Level 4/5

Number of practice sheets: 13
MathSphere
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## Notes

Children should be confident with the work in the level $3 / 4$ module before tackling the work in this module.

This is quite a straight forward module looking at sequences in different ways, except that the arithmetic is more difficult than in the Level $3 / 4$ module and the required understanding of number greater.

There is always a starting number and a rule for continuing the sequence. Sometimes children are given the sequence and need to understand the rule. Sometimes they are given the rule and need to complete the sequence.

The rules that are usually used in NCT papers are of the following forms:

Adding a constant number to the previous term.
Subtracting a constant number from the previous term.
Multiply the previous term by a fixed number, especially doubling and trebling.

Multiplying by a fixed number and adding or subtracting a constant. ( $\mathrm{Eg} \times 2$ and +5 ).

Other types occasionally appear and we have included some in this module.

Eg add 5, then add 3, then add 5, then add 3 etc.
N.B. When writing decimals it is not necessary to write trailing zeroes (i.e. 3.0 is the same as 3 and 0.0 is the same as 0 ), but it often helps if these zeroes are kept in so that children may spot a pattern more easily. Eg. 2.5, 3.0, 3.5, 4.0
Of course, sometimes they are deliberately omitted to make the children think harder!

1. Here is a sequence of numbers. Write in the missing numbers.

$$
3 \longrightarrow 11 \rightarrow 19 \rightarrow 27 \longrightarrow 35 \rightarrow \square \rightarrow \square
$$

2. Here is a sequence of numbers. Write in the missing numbers.

$$
0.6 \longrightarrow 0.9 \longrightarrow 1.2 \longrightarrow 1.5 \longrightarrow 1.8 \longrightarrow \square \rightarrow \square
$$

3. Here is a sequence of numbers. Write in the missing numbers.

The rule is "double each number"

4. Here is a sequence of numbers. Write in the missing numbers.

The rule is "triple each number and add 0.5"

$$
1 \longrightarrow 3.5 \longrightarrow 11 \rightarrow 33.5 \rightarrow \square \rightarrow \square
$$

5. Here is a sequence of numbers. Write in the missing numbers.

The rule is "multiply by 4 and subtract 3"

$$
2 \rightarrow 5 \rightarrow \square \rightarrow 65 \rightarrow 257 \rightarrow \square
$$

6. Here is a sequence of numbers. Write in the missing numbers.

The rule is "multiply by 5 and subtract 0.5 "


My friend has a boxer dog. Every time the doorbell rings, he sits in the corner!!!!!!

Why do elephants have pink toe nails?

So they can't be seen in cherry trees!


1. Here is a sequence of numbers. Write in the missing numbers.

2. Here is a sequence of numbers. Write in the missing numbers.

$$
0.2 \longrightarrow 0.7 \longrightarrow 1.2 \longrightarrow 1.7 \longrightarrow 2.2 \rightarrow \square \rightarrow \square
$$

3. Here is a sequence of numbers. Write in the missing numbers.

The rule is "halve each number"

4. Here is a sequence of numbers. Write in the missing numbers.

The rule is "multiply by 3 and subtract 5"

$$
4 \longrightarrow 7 \rightarrow 16 \rightarrow 43 \longrightarrow 124 \longrightarrow \square \rightarrow \square
$$

5. Here is a sequence of numbers. Write in the missing numbers.

The rule is "divide by 3 and add 1"

$$
123 \longrightarrow 42 \rightarrow \square \rightarrow 6 \rightarrow \square
$$

6. Here is a sequence of numbers. Write in the missing numbers.

The rule is "add 0.67 "


Mr Smith's career was in ruins. He was an archaeologist! Oh dear!

1. Here is a sequence.

## $\begin{array}{lllllll}3 & 6 & 9 & 12 & 15 & 18 & 21\end{array}$

a) Will the number 357 be in this sequence?

YES/NO
b) Explain how you worked this out.
2. Here is another sequence.

## $\begin{array}{llllllll}1 & 8 & 15 & 22 & 29 & 36 & 43 & \ldots\end{array}$

a) Will the number $\mathbf{7 0 0 2}$ be in this sequence?
b) Explain how you worked this out.
3. The rule for this sequence is "Start with 1 and 1.
To get a new term, add the two terms before it."

## $\begin{array}{llllllll}1 & 1 & 2 & 3 & 5 & 8 & 13 & 21\end{array}$

a) What is the $10^{\text {th }}$ term in this sequence?
b) $\mathbf{2 1}$ is divisible by 7 .

What is the next number in this sequence that is divisible by 7 ?

Did you hear about the man who stole a calendar?

He got twelve months!


1. The rule for this sequence is
"Start with 3 and 4.
To get a new term, add the two terms before it."

## $\begin{array}{llllll}3 & 4 & 7 & 11 & 18 & 29\end{array}$

a) What is the $10^{\text {th }}$ term in this sequence?

b) Work out the first 20 terms of this sequence, writing down only the last digit each time, like this:

$$
\begin{array}{llllll}
3 & 4 & 7 & 1 & 8 & 9
\end{array}
$$

c) Now explain why no multiple of 5 will ever appear in the sequence $\begin{array}{llllll}3 & 4 & 7 & 11 & 18 & 29\end{array}$
no matter how many terms you work out.
2. Here is a sequence:

## $400340280 \quad 220$...

a) What is the rule for this sequence?
b) What are the first two numbers in this sequence that are below zero?


1. Here is a sequence. Each number is in a square.




a) Which number will be in the $\mathbf{2 6}{ }^{\text {th }}$ square?

b) Which number will be in the $500^{\text {th }}$ square?
c) How did you get your answers to parts a) and b) ?
d) What is the rule for this sequence?
2. Here is a sequence. Look carefully at the way it works.


Put the missing numbers in the empty boxes.
3. Jeremy makes a sequence by starting with the number 0.5 and adding 0.6 each time.
Put the first numbers of the sequence in the boxes:


1. Here is a sequence. Each number is in a square.

| 0.3 | 0.5 | 0.7 | 0.9 | 1.1 | 1.3 | 1.5 | 1.7 | 1.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 \text { square }$ | 2 | square 3 | square | square | $\begin{gathered} \text { square } \\ \hline \end{gathered}$ | $\begin{aligned} & \text { square } \\ & 7 \end{aligned}$ | $\begin{gathered} \text { uare } \\ 8 \end{gathered}$ | 9 ${ }^{\text {a }}$ |

a) Which number will be in the $10^{\text {th }}$ square?

b) Which number will be in the $50^{\text {th }}$ square?

c) How did you get your answer to part b) ?
d) What is the rule for this sequence?
2. Here is a sequence. Look carefully at the way it works.

a) Put the missing numbers in the empty boxes.
b) What are the rules for this sequence?

Last time I went on holiday for a fortnight it only rained twice - once for seven days and once for a week!

1. The numbers in the following sequence increase by the same difference each time.
Put the missing numbers in the boxes.
-1.4

0.1
2. The numbers in the following sequence increase by the same difference each time.
Put the missing numbers in the boxes.
0.06
$\square$
$\square$ 0.15 $\square$
$\square$
3. Peter makes a sequence by starting with 8.4 and subtracting 2.2 each time.
Put the missing numbers in his boxes.
8.4
$\square$

$\square$
4. Put in the missing numbers in this sequence.
0.34
0.46
0.58 $\square$
$\square$ 0.94
5. Put the missing numbers in this sequence.
$8.66 \square$
6.22
5.00 $\square$ 2.56
6. Put in the missing numbers in this sequence.
$\square$
0.51 .0
2.0 $\square$ 8.0

And there's a nice easy one to finish with.

Geraldine wanted to sleep like a log so she put her bed in the wood shed!

1. The numbers in the following sequence increase by the same difference each time.
Put the missing numbers in the boxes.

2. The numbers in the following sequence increase by the same difference each time.
Put the missing numbers in the boxes.
0.14

$\square$ 0.74

$\square$
3. Jo makes a sequence by starting with 9.6 and subtracting 1.4 each time.
Put the missing numbers in his boxes.
9.6

4. Put in the missing numbers in this sequence.
0.88
$1.00 \quad 1.12$

1.48
5. Put in the missing numbers in this sequence.

6. Put in the missing numbers in this sequence.
$\square$
0.4
0.8
1.6 $\square$ 6.4

Did you hear about the man who was so rich he sent his grass away to be cut!!

1. Put the missing numbers on this number line.

2. Put the missing numbers on this number line.

3. Put the missing numbers on this number line.

4. Put the missing numbers on this number line.

5. In the following sequence each number is double the one before.

Fill in the missing numbers.

$\begin{array}{lll}0.7 & 1.4 & 2.8\end{array}$
5.6
6. In the following sequence each number is double the one before. Fill in the missing numbers.
$\square-1.2 \quad-2.4 \quad-4.8 \quad \square$
7. In the following sequence each number is double the one before. Fill in the missing numbers.

$\square$|  | $\square$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 0.09 | 0.36 | $\square$ |  |

1. Put the missing numbers on this number line.

2. Put the missing numbers on this number line.

3. Put the missing numbers on this number line.

4. Put the missing numbers on this number line.

5. In the following sequence each number is the one before multiplied by 4 . Fill in the missing numbers.
$\square 0.20 \quad 0.80 \quad \square \quad \square$
6. In the following sequence each number is double the one before plus 0.1.
Fill in the missing numbers.

|  |  |  |  |  |  | 2.3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 4.7 | 9.5 | $\square$ |  |  |  |

7. In the following sequence each number is triple the one before. Fill in the missing numbers.

$\square$|  | -2.7 | -8.1 | -24.3 |
| :--- | :--- | :--- | :--- |$\square$

Can you describe the rules for the following sequences. Remember to give the starting number and the rule for continuing the sequence.

Yes, of course we can!

Example: Find the rule for this sequence.
0.2
0.4
0.8
1.6
3.2
6.4

Rule: Start with the number 0.2 and double each term.
Now you try.
$\begin{array}{lllllll}1 . & 0.4 & 1.2 & 3.6 & 10.8 & 32.4 & 97.2\end{array}$
$\begin{array}{llllllll}2 . & 7.4 & 7.2 & 7.0 & 6.8 & 6.6 & 6.4 & 6.2\end{array}$
3. $\quad-6$
$\begin{array}{llllllll}4 . & 0.3 & 0.7 & 1.1 & 1.5 & 1.9 & 2.3 & 2.7\end{array}$
$\begin{array}{lllllll}5 . & 15 & 7.5 & 3.75 & 1.875 & 0.9375 & 0.46875\end{array}$
6. $\quad 32768 \quad 163848192 \quad 40962048$

Give the next two numbers in each sequence:
7. 224
$6 \quad 10$ $\square$
$\square$
8. $25 \begin{array}{llll} & 5 & 11 & 23\end{array}$

9. $\quad 10 \quad 35 \quad 110 \quad 335$
 sequence.

I hope you're not going to sneak off for a quick snack, Multy!
Example: Find the rule for this sequence.

35
$9 \quad 17$
33
65

Rule: Start with the number 3, then multiply by 2 and subtract 1.
Now you try.
$\begin{array}{llllllll}1 . & 0.8 & 1.4 & 2.0 & 2.6 & 3.2 & 3.8 & 4.4\end{array}$
2. $\begin{array}{llllllll}0 & -1 & -3 & -7 & -15 & -31 & -63\end{array}$
$\begin{array}{lllllll}3 . & 100 & 99.9 & 99.8 & 99.7 & 99.6 & 99.5\end{array}$
$\begin{array}{lllllllll}4 . & 1 & 4 & 9 & 16 & 25 & 36 & 49 & 64\end{array}$
$\begin{array}{llllllll}\text { 5. } & 0.1 & 0.4 & 0.9 & 1.6 & 2.5 & 3.6 & 4.9\end{array}$
6. $\quad \begin{array}{llllllll}3.2 & 2.4 & 1.6 & 0.8 & 0.0 & -0.8 & -1.6 & -2.4\end{array}$

Give the next two terms in each sequence:
7. 5
8. $12 \quad 23 \quad 45 \quad 89 \quad \square \quad \square$
9. $\begin{array}{llllllll}\mathrm{S} & \mathrm{M} & \mathbf{T} & \mathrm{W} & \mathbf{T} & \square\end{array}$

Fun Sequences
Here are some fun sequences you could investigate.


1. Begin with 12. Keep dividing by 2 and adding 4. A calculator may be useful.
2. Using the same idea, make up a sequence yourself that behaves in the same way.
3. Can you use the same idea to produce this sequence?

$$
\begin{array}{llllllll}
6 & 6 & 6 & 6 & 6 & 6 & 6 & \ldots
\end{array}
$$

4. Using an idea on another page of this module, can you produce the following sequence?

$$
2,4,8,6,2,4,8,6,2,4,8,6, \ldots
$$

5. Can you produce the following sequence?

$$
1,-1,1,-1,1,-1,1,-1,1,-1, \ldots
$$

6. What are the rules for this sequence?

$$
6,8,11,13,16,18,21,23,26, \ldots
$$

7. How do you make this sequence?

$$
1,3,6,10,15,21,28,36, \ldots
$$

8. Where does this sequence come from?

## Answers

## Page 3

1. 43,51
2. 2.1, 2.4
3. $-112,-224$
4. $101,303.5$
5. 17,1025
6. $47,5859.5$

## Page 4

1. 69,81
2. $2.7,3.2$
3. $4.5,2.25$
4. 367,1096
5. 15,3
6. $4.41,5.08$

## Page 5

1. a) Yes
b) This is a sequence of multiples of 3 . The digits 3,5 and 7 total 15 , so 357 is a multiple of 3 .
2. a) No
b) Each term is one more than a multiple of 7.7002 is 2 more than a multiple of 7 .
3. a) 55
b) 987

Page 6

1. |  | a) | 199 | b) | 3 | 4 | 7 | 1 | 8 | 9 | 7 | 6 | 3 | 9 | 2 | 1 | 3 | 4 | 7 | 1 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

c) The last digits repeat the pattern $3,4,7,1,8,9,7,6,3,9,2,1$ for ever. To be a multiple of 5 the last digit must be a 0 or a 5 , which is not in the sequence.
2.
a) Start with 400 and subtract 60
b) -20 and -80

## Page 7

1. 

a) 2.6
b) 50
c) Divide the number of the square by 10 .
d) Start with 0.1 and add 0.1 .
2. -4 (Bottom) and 3 (Top)
3. $0.5,1.1,1.7,2.3,2.9,3.5,4.1$

## Page 8

1. a) 2.1
b) 10.1
c) Divide the number of the square by 5 and add 0.1 .
d) Start with 0.3 and add 0.2 .
2. a) $-1.2,-1.0,-0.6,-0.4,0.0$
b) Start with -3 . Repeat: add 0.2, add 0.4.

## Answers (Contd)

## Page 9

1. $-1.1,-0.8,-0.5,-0.2$,
2. $0.09,0.12$,
$0.18, \quad 0.21$
3. 6.2,
$1.8,-0.4,-2.6$
4. $0.70,0.82$
5. 7.44 ,
3.78
6. 0.25 ,
4.0

Page 10

1. $-0.3,0.1,0.5,0.9$
2. $0.34,0.54$,
$0.94,1.14$
3. $8.2 \quad 5.4,4.0,2.6$
4. $1.24,1.36$
5. 19.8,
19.2
6. $0.2,3.2$

## Page 11

1. $0.8,0.9,1.0,1.1$
2. $3.98,3.99,4.00,4.01$,
3. $-0.4,-0.2$,
$0.2, \quad 0.4$
4. $0.175,0.35,11.2$
5. $-0.3,-0.6 \quad-9.6$
6. $0.0225,0.045,0.72$

## Page 12

1. 6.0, 6.2, 6.6, 6.8 2. 32.8, 33.0, 33.2, 33.4
2. 4.76 ,
4.78, 4.79,
4.81
3. $-4.0,-3.5,-2.5,-1.5$
4. $0.0125,0.05$
$3.20,12.80$
5. $0.2,0.5$
19.1
6. $-0.3,-0.9$
$-72.9$

## Page 13

1. Start at 0.4 and triple $(\times 3)$
2. Start at -6 and add 0.5
3. Start at 15 and divide by 2
4. 16,26
5. 47,95
6. 1010 and 3035

## Page 14

1. Start at 0.8 and add 0.6
2. Start at 0 and multiply by 2 and subtract 1
3. Start at 100 and subtract 0.1
4. Start at 1 and then either square numbers or add $3,5,7$ etc
5. Same as 4. then divide by 10
6. Start at 3.2 and subtract 0.8
7. 25 and 40
8. 177 and 353
9. F S (Days of the week)

## Page 15

1. $12,10,9,8.5,8.25,8.125, \ldots$

This sequence approaches 8 .
2. To make a similar series keep dividing and adding, but you may well find yourself going into the negatives. All good fun, really!
3. Start with 6 , divide by 2 and add 3 . Or divide by 3 and add 4 . Or divide by .
4. Start with 2 and keep doubling, but only write down the last digit of each term.
5. Start with 1 and keep multiplying by -1 . Any other suggestions?
6. Start with 6 then repeat: Add 2, Add 3.
7. These are the triangle numbers. Begin with 1 and add 2 , add 3 , etc.

The formula is $\mathrm{n}(\mathrm{n}+1) / 2$ where n is the number of the term.
8. Days in the months of the year. Just to show there are other ways of making sequences.

